Strategic Pressure Measurement System Characterization of the Mars Entry Atmospheric Data System

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Outline

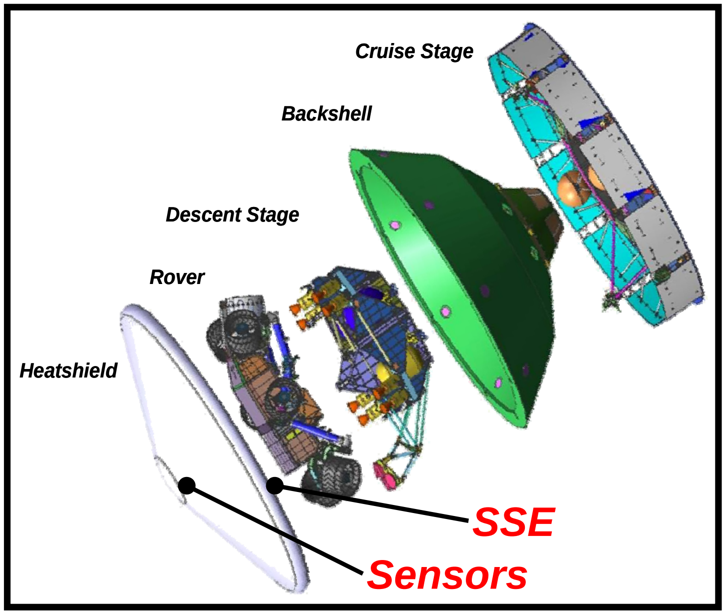
- Mars Entry Atmospheric Data System (MEADS) Requirements
- Characterization Challenges
- Response Surface Methodology
  - Mathematical Model
  - Experimental Design and Execution Protocol
  - Design Performance
  - Uncertainty Quantification
- Integration into Flight Data Algorithm
- Summary of Approach

MEADS Objectives and Requirements

- **Mars Science Laboratory (MSL) entry descent and landing instrumentation (MEDLI), entry atmospheric data system**
  - **Estimate vehicle attitude and atmospheric density from pressure measurements at 7 locations on the heat shield**
  - **Improve EDL simulation for robust Mars entry systems**
- **Defendable uncertainty in flight parameters relies on adequate measurement system characterization over extreme environments**
- **Uncertainty goal of 1% of reading through the range of 0.12 – 5.0 psia**
- **Characterization/Calibration deliverable products**
  - **Mathematical model to estimate flight pressure**
  - **Uncertainty estimates throughout the flight trajectory**

Deliverables are measurement system knowledge, not calibration data

System Description and Characterization Space Definition



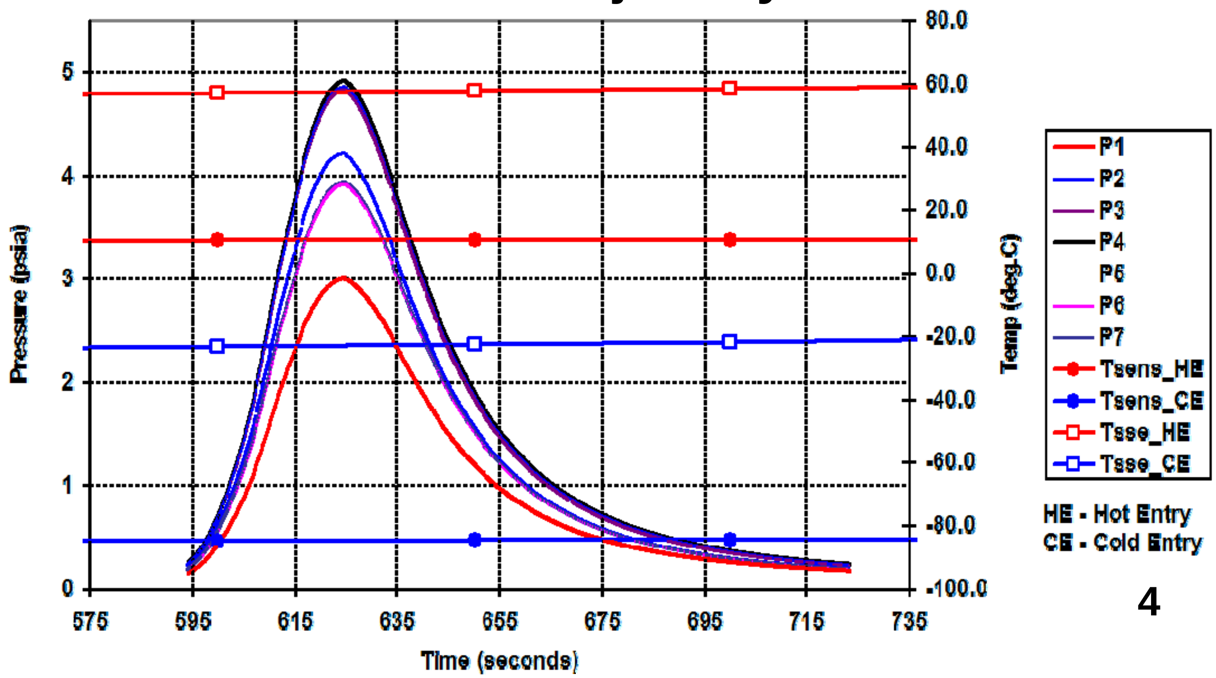
Measurement System Components

- 7 pressure transducers (sensors)
- Signal support electronics (SSE)
- Temperatures vary between SSE and sensors locations (start and entry)
- Pressure varies across port locations

Characterization Space

Pressure	0 to 5 psia
Temp. Sensor	-90 to +30 deg.C
Temp. SSE	-35 to +65 deg.C

Predicted Trajectory



Characterization Challenges

- **How should we characterize (calibrate) the measurement system to ensure defendable uncertainty estimates to meet research objectives?**
- **What is our modeling strategy?**
  - **How can we test if it is adequate?**
  - **How will the model be integrated with the flight data algorithm?**
  - **How do we quantify uncertainty over the environment?**
- **How do we build a test matrix (design) to support our model?**
  - **Which design points to choose – locations?**
  - **How many design points – data volume?**
  - **What is the quality of the design – performance?**

Response Surface Methodology (RSM)

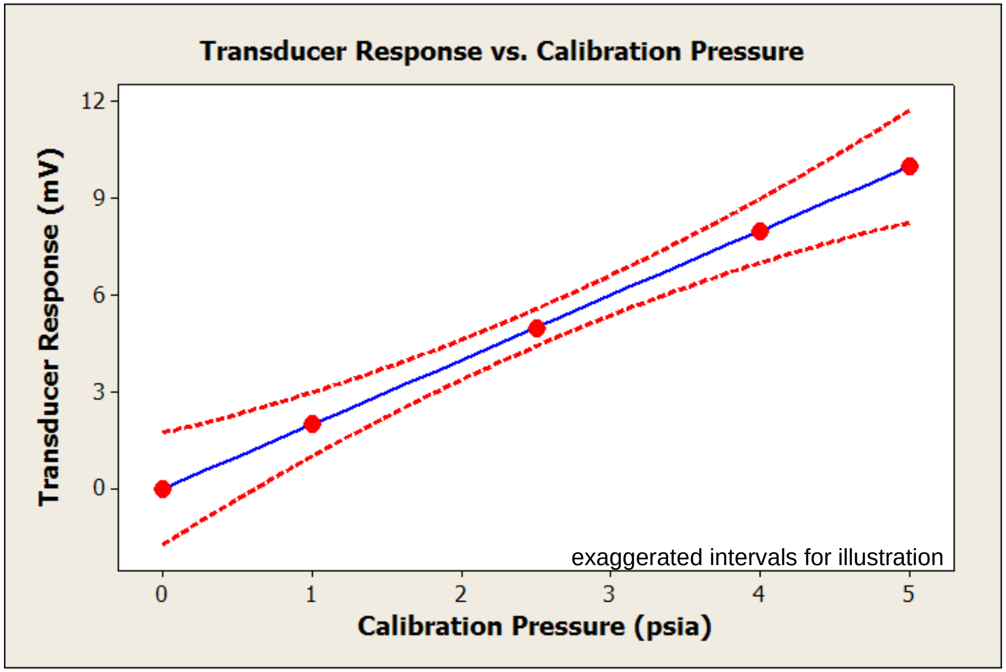
- An extension of Statistical Design of Experiments (DOE)
- Developed in the 1950’s in the chemical industry
  – 50+ year successful track-record in industry and science
  – RSM-based calibration has been performed at Langley since 1999

Features of the Methodology

- Scientifically disciplined and mathematically rigorous decision-making framework to design, execute, and analyze experiments
- Systems engineering perspective - emphasizes integration
  – Efficient, strategic, tactical, objective, defendable
  – Not a replacement for good science and engineering
- Fundamental Principles
  – Design Efficiency
  – Randomization, Replication, Blocking

Model and Design – Simple Example

- **Goal: Minimize the width of the confidence interval = lower uncertainty**
- **Where should we set the pressure levels for calibration?**



- **Maximize *Sxx* = Spread the points apart**
- **Optimal: equal replication at 0 and 5**

Calibration Model

$$V = f(P) + \varepsilon$$

$$V = \beta_0 + \beta_1 P + \varepsilon$$

zero intercept — **sensitivity (slope)** — **error (noise)**

Model Predictions

$$\hat{V} = \hat{\beta}_0 + \hat{\beta}_1 P$$

Confidence Interval on Prediction

$$\hat{V} \pm t_{\alpha/2,df}\,\hat{\sigma}_\varepsilon \sqrt{\frac{1}{n} + \frac{\left(P - \overline{P}\right)^2}{S_{xx}}}$$

$$S_{xx} = \sum_{i=1}^{n} (P_i - \overline{P})^2$$

Mathematical Model for MEADS

- **Consider a second-order Taylor series expansion in 3 factors**

zero intercept adjustments as a function of temperature

$$V = \beta_0 + \beta_1 P + \beta_2 T_{sensor} + \beta_3 T_{SSE}$$

second-order effect of pressure (non-linearity)

$$+\beta_{11} P^2 + \beta_{22} T_{sensor}^2 + \beta_{33} T_{SSE}^2$$

second-order effects of temperature on intercept

$$+\beta_{12} P x T_{sensor} + \beta_{13} P x T_{SSE}$$

sensitivity adjustments as a function of temperature

$$+\beta_{23} T_{sensor} x T_{SSE} + \varepsilon$$

assumed negligible based on system knowledge

β's are the calibration coefficients

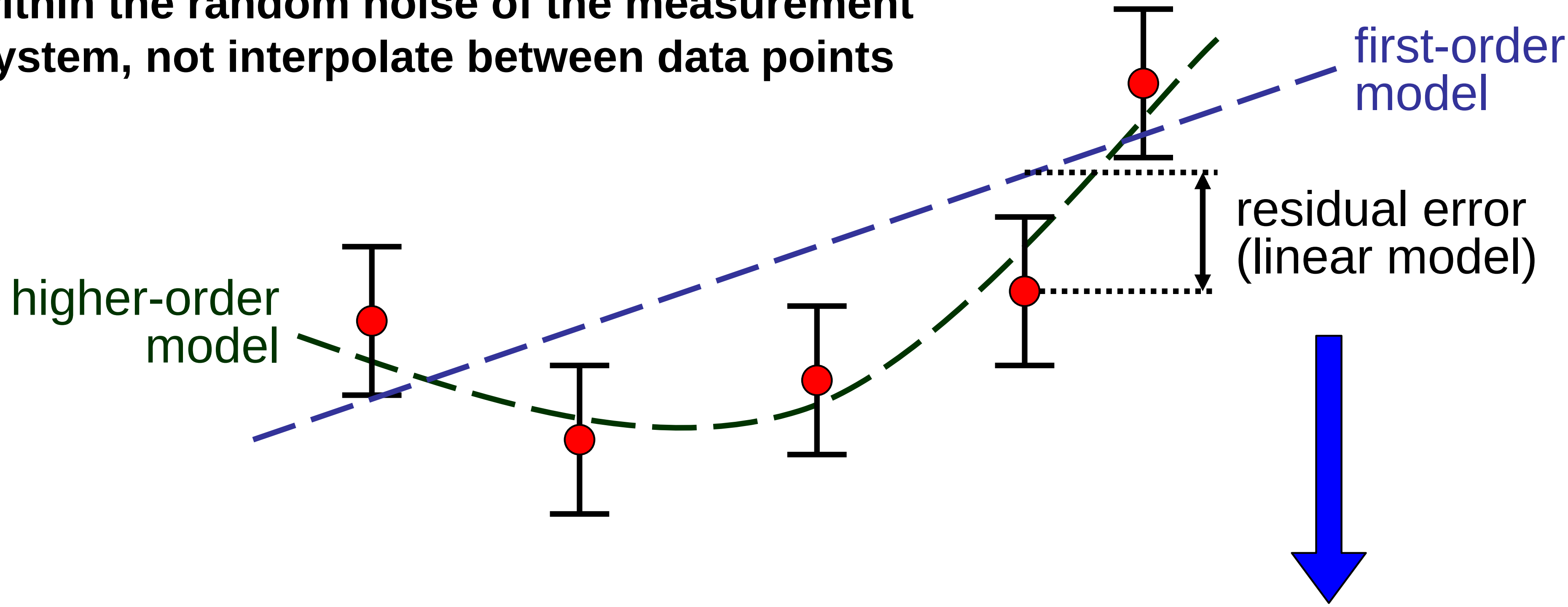
ε is the experimental error

final model contains terms that are statistically significant

Testing Model Adequacy

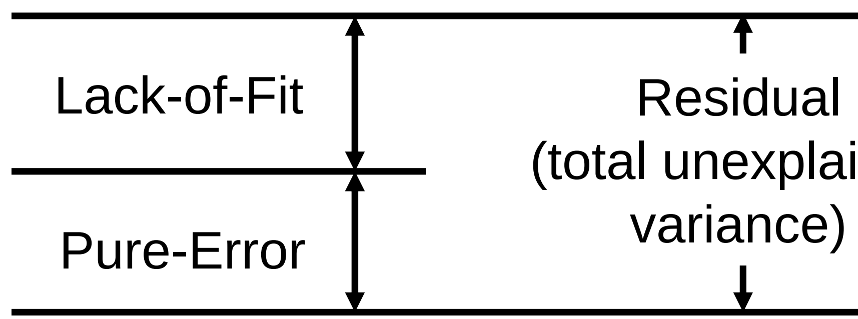
What is a good (adequate) model?

- **It is able to reproduce the experimental data within the random noise of the measurement system, not interpolate between data points**



<u>Analysis of Unexplained Variance</u>

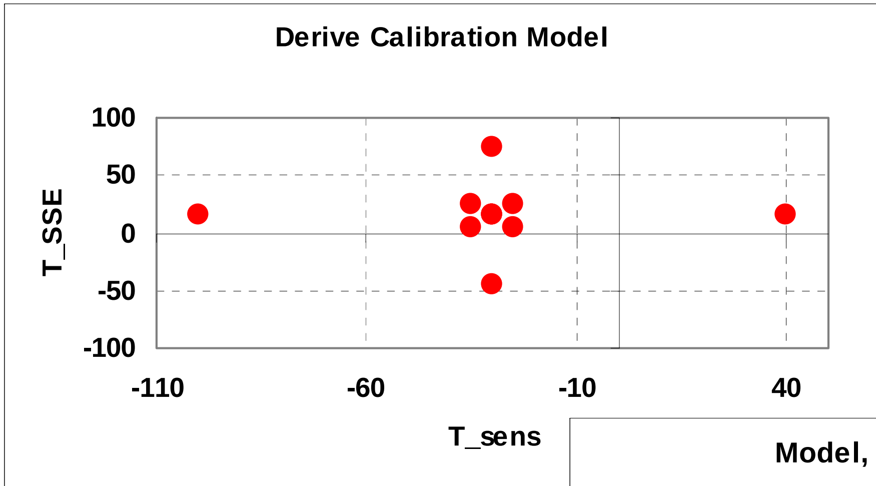
- **Errors are partitioned into model lack-of-fit and experimental noise**
- **Replication provides pure-error**

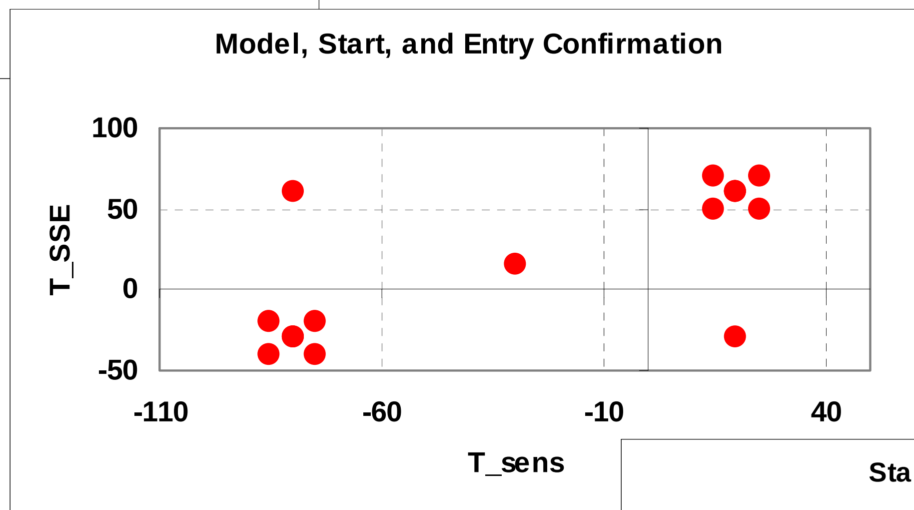


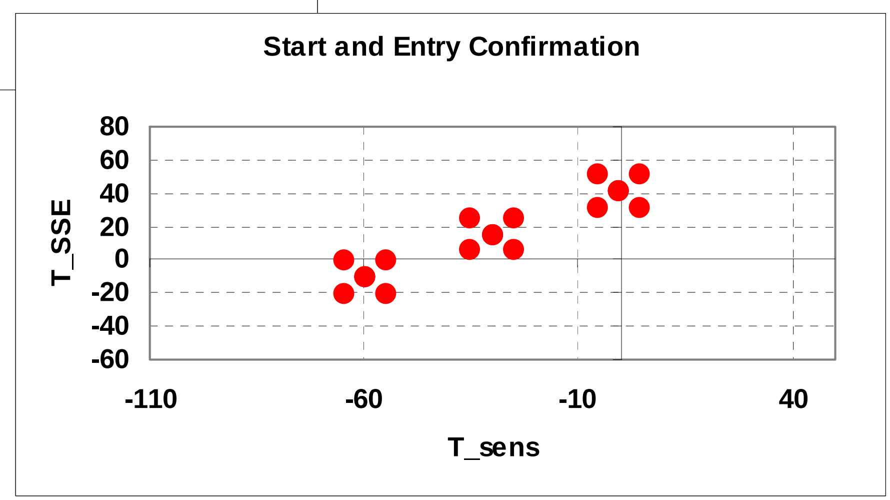
Experimental Design and Execution

<u>Temperature Design Space</u>

- **Design is partitioned into 3 blocks to isolate day-to-day variability**
  - **Model development**
  - **Confirmation (test the model)**

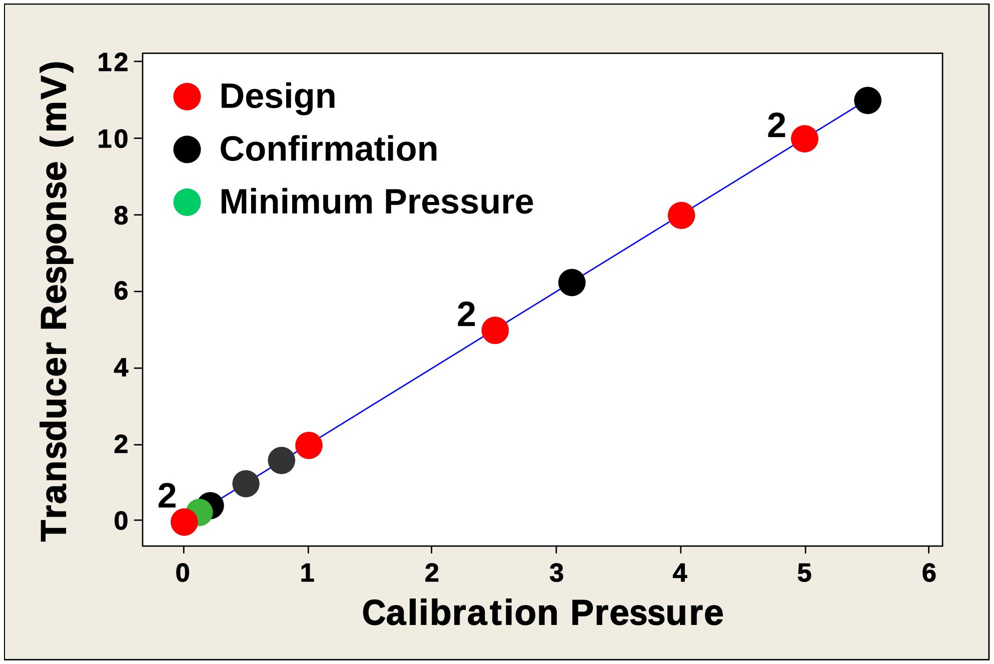






- **Temperatures are set in random order**
- **Replication throughout design space**
- **Comprehensive assessment of the measurement system performance over predicted trajectories**

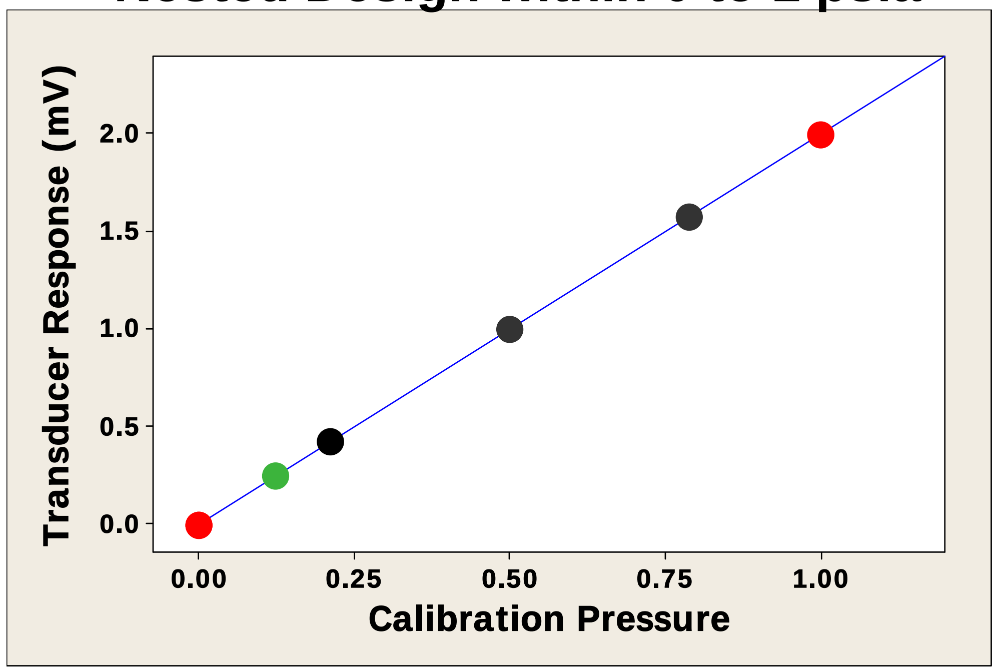
Pressure Design Space



Distribution of Information (# of pts)

- 0-5 psia calibration (8)
- 0-1 psia low-end calib./conf. (3)
- 0.12 psia (850 pa) confirmation (1)
- 5.5 psia 3σ max confirmation (1)
- random conf. at mid-range (1)

Nested Design within 0 to 1 psia

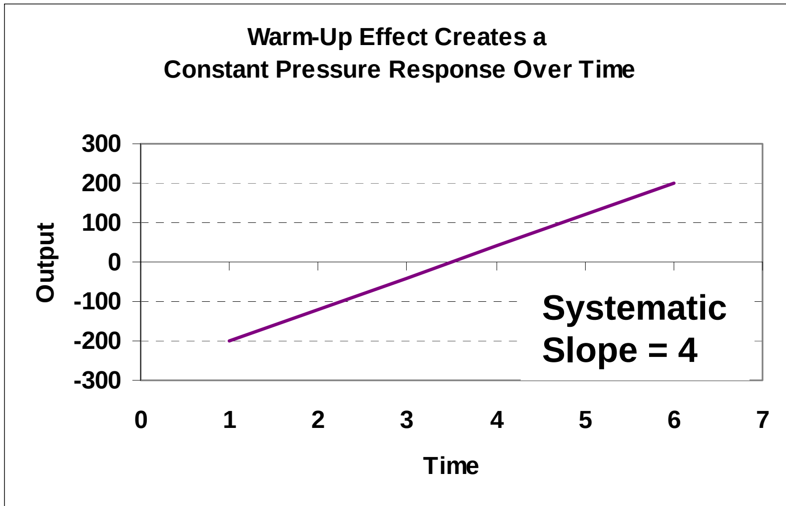


Nested randomization

- Once a temperature combination is set, pressure settings are completely randomized

Numbers indicate replicated design points

Why a Randomized Point Ordering?

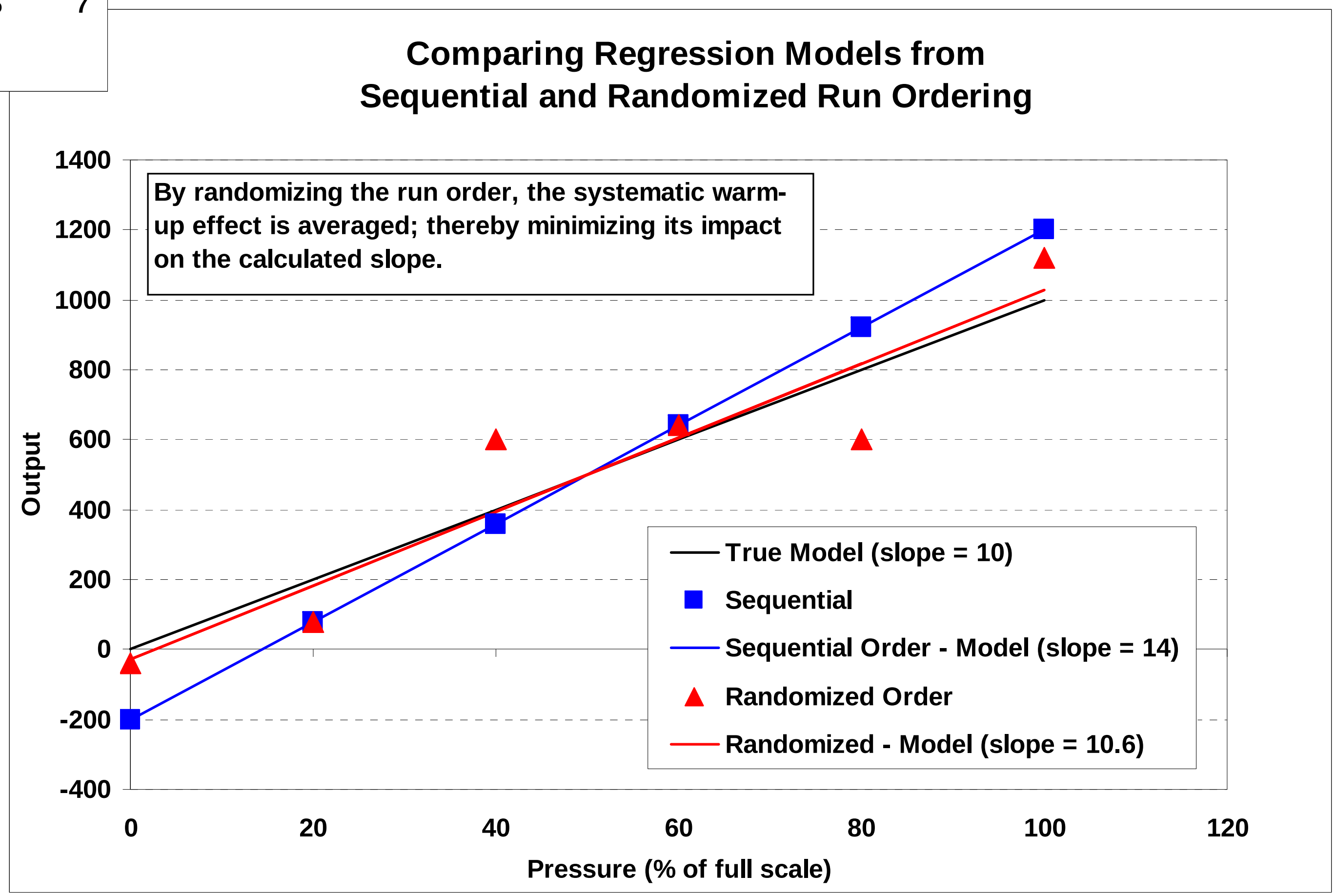


- We want to model the measurement system, not the calibration apparatus
- Randomization defends against unknown systematic variation correlated with calibration factors (pressure, temperature)

Superimposing Systematic

Time	P	Sequential
1	0	-200
2	20	80
3	40	360
4	60	640
5	80	920
6	100	1200

Time	P	Random
1	80	600
2	20	80
3	0	-40
4	60	640
5	100	1120
6	40	600



exaggerated systematic variation for illustration

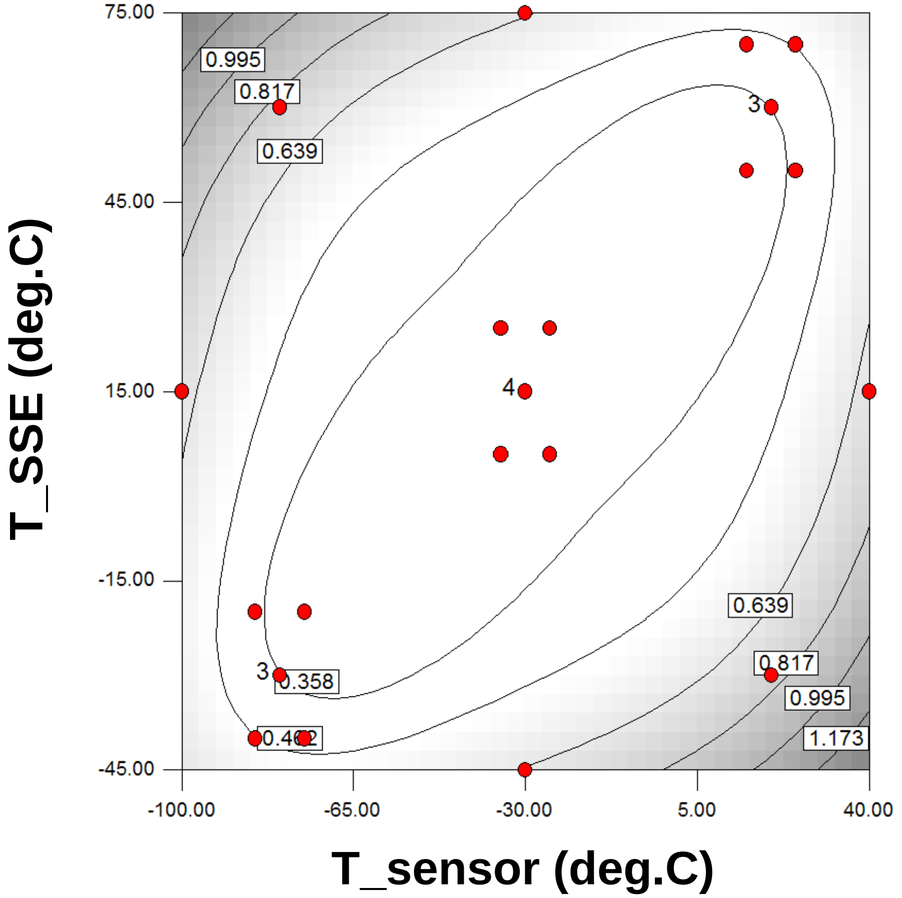
Design Features

Factors	**3 factors: T(sensor), T(SSE), Pressure**
Factor Levels	**3 of T(sensor) and T(SSE), 5 of Pressure**
Design Construction	**Axial points in Temp, Nested in Pressure**
Model Supported	**Second-order model (9 terms), without T(sens) x T(SSE)**
Unique points	**25**
Lack of Fit	**16 degrees of freedom (df), detect 4th order in Pressure**
Replication	**5 reps of Temperature, 4 pure-error df** **3 reps of pressure within each temp., 30 pure-error df**
Randomization	**Nested restricted randomization,** **Pressure randomized within randomized Temperatures**
Blocking	**one block defined as an approximate 24 hour day**
Confirmation Points	**5 temperature combinations, 6 levels of Pressure**

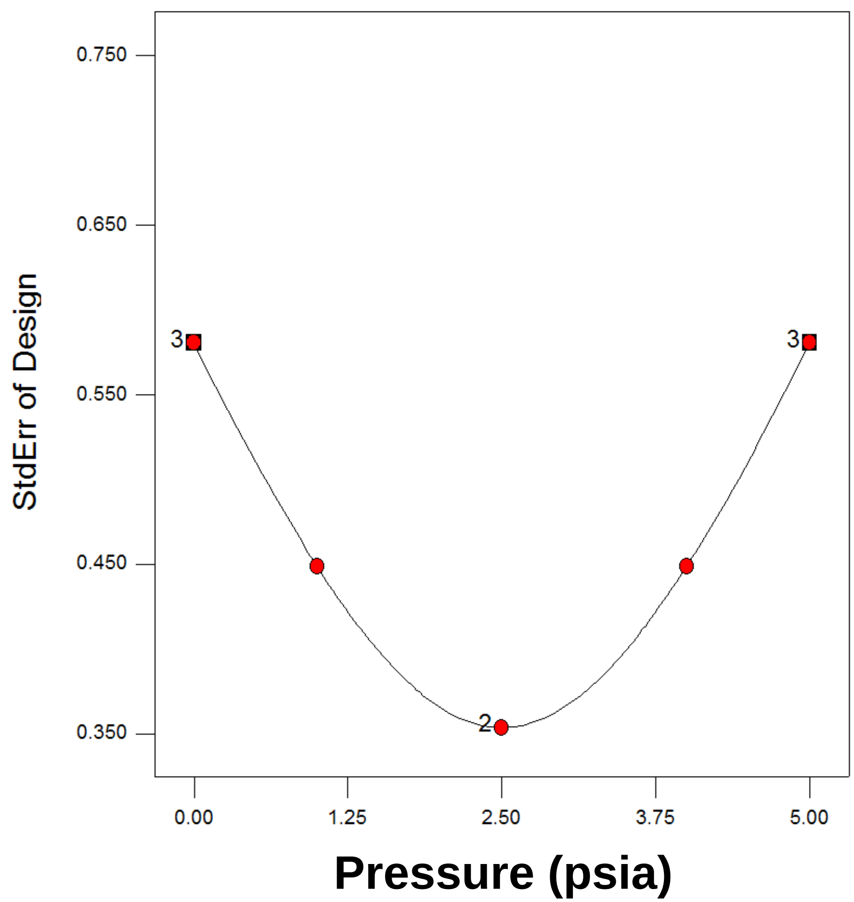
Design Performance - Prediction Variance

- Scaled prediction variance is a multiple of the pure-error
- Contours depend on design and model, not the experimental data
- Lower prediction variance = lower measurement uncertainty

Temperature Space



Pressure Space



Note: TxP interactions space is omitted for clarity

Uncertainty Quantification

- **How do we quantify measurement system uncertainty and mission specific performance? Consider 2 components of uncertainty:**
  1. **Pressure Measurement Uncertainty**
  2. **Calibration Model Stability**

<u>Measurement Uncertainty</u>

- **simple: pressure measurement uncertainty over the calibration space**
  - **metric: distribution of 3σ prediction intervals (PI)**
- **mission specific: uncertainty along the predicted trajectory**
  - **metric: 3σ PI along the trajectory**

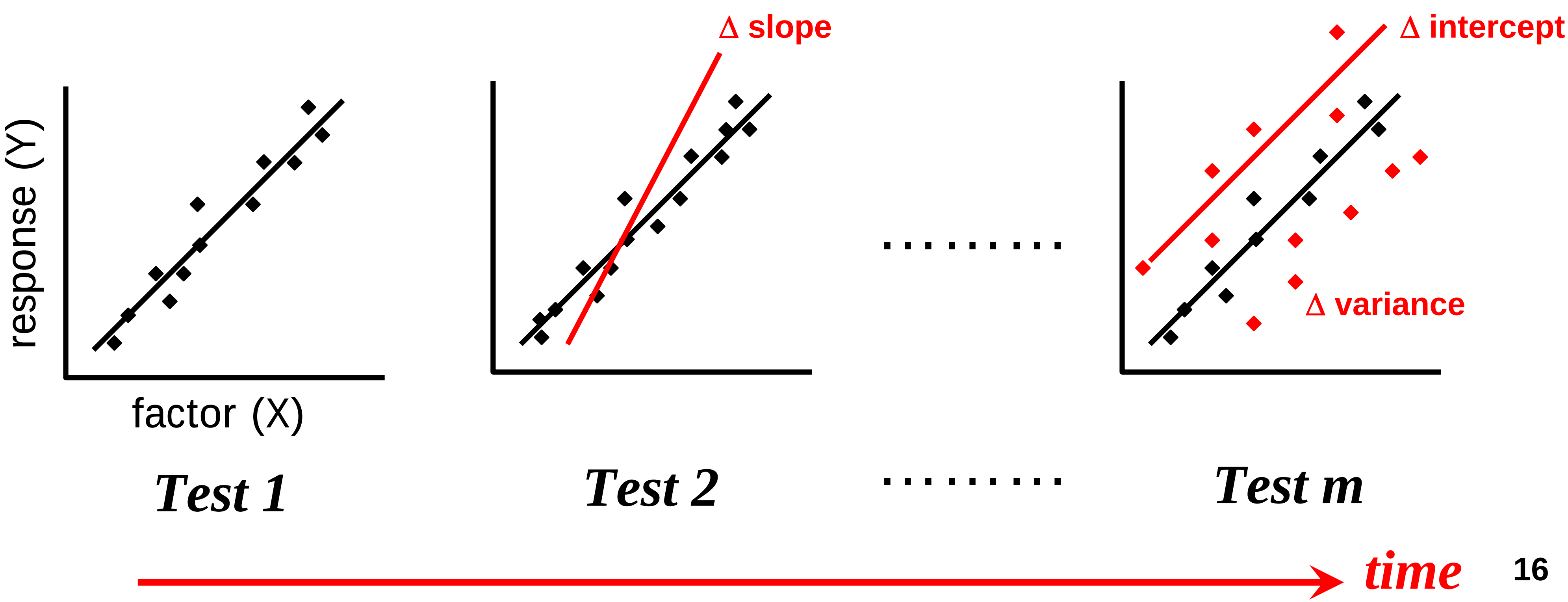
<u>Stability of Calibration Model</u>

- **Over repeated environmental excursions (vibration, thermal-vacuum, out-gassing, microbial reduction)**
- **periodic stability tests quantify the variability in model coefficients**
  - **metric: 3σ interval of each calibration coefficient**

Stability Monitoring

- **Stability tests are performed periodically at room temperature to monitor the calibration model coefficients, not the raw data**

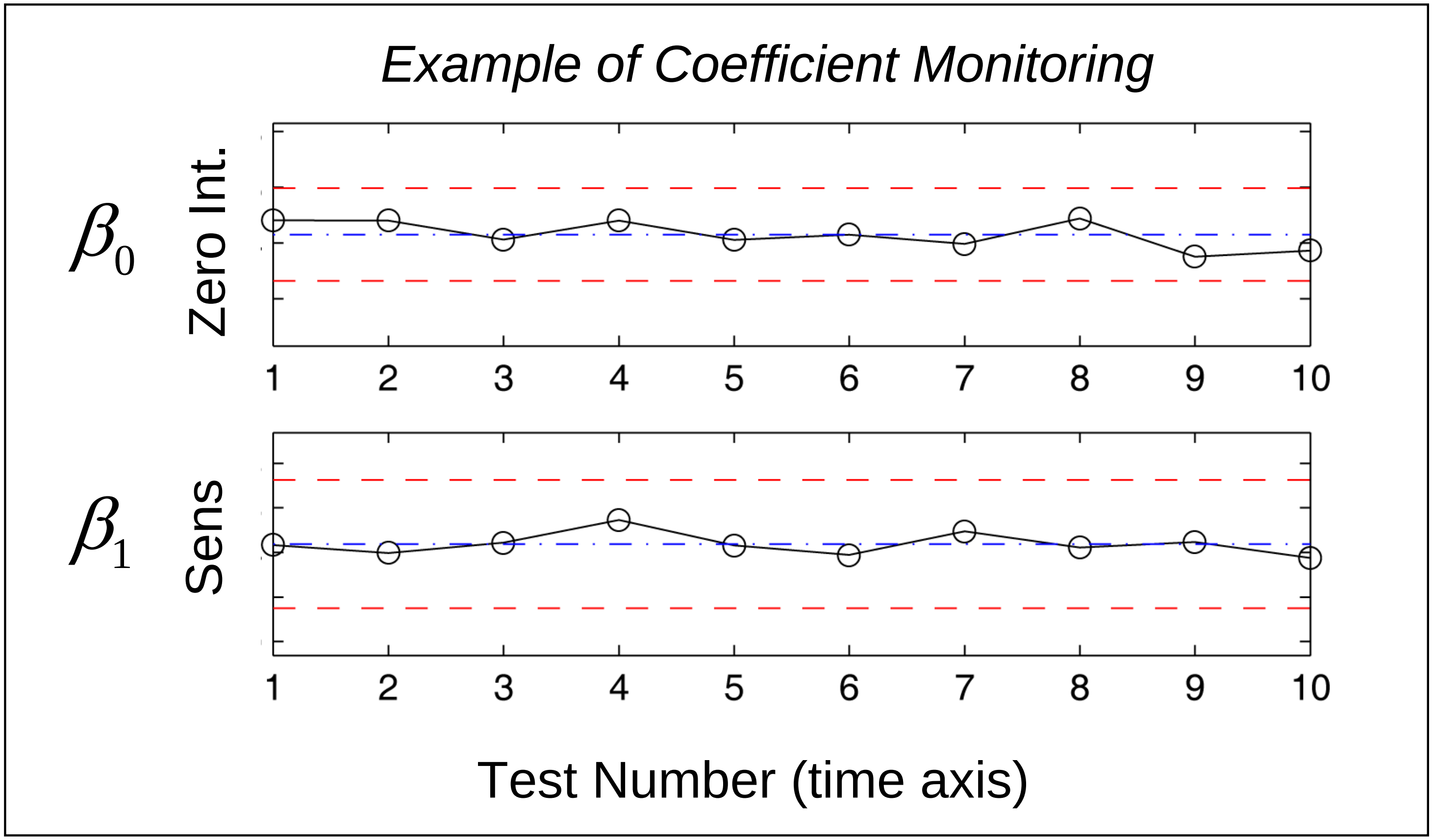
$$
\begin{aligned}
V = {} & \boxed{\beta_0} + \boxed{\beta_1} P + \beta_2 T_{sensor} + \beta_3 T_{SSE} + \boxed{\beta_{11}} P^2 + \beta_{22} T_{sensor}^2 + \beta_{33} T_{SSE}^2 \\
& + \beta_{12} P x T_{sensor} + \beta_{13} P x T_{SSE} + \beta_{23} T_{sensor} x T_{SSE} + \boxed{\varepsilon}
\end{aligned}
$$



Conceptual Simple Linear Example

Analysis of Stability

- **Control charts graphically monitor the model stability**
  - **red lines indicate expected level of *common cause variation***
    - **estimated from the replicated stability tests within the initial baseline calibration**
  - **a value beyond the red line indicates a statistically significant change in a calibration coefficient, *signal* – *Decision Point***



Integration into Flight Data Algorithm

- **A forward model is developed for each pressure channel**

$$V = f\left(P, T_{sensor}, T_{SSE}\right)$$

- **An inverse model is used for flight data reduction, with uncertainty**

$$\hat{P} = f\left(V, T_{sensor}, T_{SSE}\right) \pm \text{uncertainty interval}$$

Estimate of *P* uncorrected for interactions that are a function of *P*

$$\hat{P}_{uncorr} = \frac{V - \left(\hat{\beta}_0 + \hat{\beta}_2 T_{sens} + \hat{\beta}_3 T_{SSE} + \hat{\beta}_{22} T_{sens}^2 + \hat{\beta}_{33} T_{SSE}^2 + \hat{\beta}_{23} T_{sens} x T_{SSE}\right)}{\hat{\beta}_1}$$

Interactions that are a function of *P*

$$\sum \text{Interactions(P)} = \frac{\hat{\beta}_{11}}{\hat{\beta}_1}\hat{P}^2 + \frac{\hat{\beta}_{12}}{\hat{\beta}_1}\hat{P} x T_{sens} + \frac{\hat{\beta}_{13}}{\hat{\beta}_1}\hat{P} x T_{SSE}$$

Solve iteratively to converge on a point estimate of *P*

$$\hat{P} = \hat{P}_{uncorr} - \sum \text{Interactions(P)}$$

Pressure Uncertainty

Pressure uncertainty depends on the following components

- **Location in design space (trajectory):** $\hat{\mathbf{x}} = f\left(\hat{P}, T_{sensor}, T_{SSE}\right)$
- **Calibration design matrix, expanded in model form:** $\mathbf{X}$
- **Covariance matrix of response observations:** $\operatorname{cov}(\mathbf{V}) = \mathbf{\Sigma}$
- **Variance of model coefficients:** $\operatorname{var}(\hat{\boldsymbol{\beta}}) = \left(\mathbf{X}'\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}$
- **Vector of partial derivatives with respect to each estimated coefficient:**

$$\partial\mathbf{g} = \partial\hat{P}\left(\hat{\mathbf{x}}\right)$$

- **Confidence Interval:**

$$\left|P - \hat{P}\right| \approx \left[\partial\mathbf{g}^{T}\left(\mathbf{X}'\mathbf{\Sigma}^{-1}\mathbf{X}\right)^{-1}\partial\mathbf{g}\right]^{(1/2)}\left(t_{df,\alpha/2}\right)$$

Summary of Approach

- Characterization planning, design, modeling, and uncertainty strategically support defendable uncertainty estimates of flight parameters; satisfying the science objectives
- Design performance is quantitatively assessed before execution
- Execution incorporates strategic and tactical techniques
  - estimates experimental error (system noise), pure-error
  - defends against systematic variation in the apparatus
  - efficiently collect information over characterization space
- Modeling and uncertainty analysis
  - builds and tests adequate mathematical models
  - provides uncertainty estimates over the trajectory
- Provides a general framework applicable to measurement systems

Contacts and References

Contacts for more information:

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Michelle Munk (michelle.m.munk@nasa.gov)

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